**Supplementary Materials**

# Model predictive tracking control based on adaptive sliding mode constraints for unmanned underwater vehicles

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The force and moment generated by the thrusters in a specific motion state can be derived from the Equation (1). Here, $X$, $Y$ , and $Z$ represent the force along the $Ox$-axis, $Oy$-axis, and $Oz$-axis, respectively. $K$, $M$, and $N$ denote the moment around the $Ox$-axis, $Oy$-axis, and $Oz$-axis, respectively. The other relevant parameters are presented in Supplementary Tables 1 and 2. And the parameters of the controller used in the Section 4 are presented in Supplementary Table 3.

$$
\begin{cases}
X = m[\dot{u} - vr + wq] - x_g(q^2 + r^2) + y_g(pq - \dot{r}) + z_g(pr + \dot{q}) + X_u u + X_{|u|u}|u|u \\
Y = m[\dot{v} - wp + ur] - y_g(r^2 + q^2) + z_g(pr - \dot{p}) + x_g(qp + \dot{r}) + Y_v v + Y_{|v|v}|v|v \\
Z = m[\dot{w} - uq + vp] - z_g(p^2 + q^2) + x_g(rp - \dot{q}) + y_g(rp + \dot{p}) + z_w w + Z_{|w|w}|w|w \\
K = I_x \dot{p} + (I_z - I_y)qr + K_p p + K_{|p|p}|p|p + m[y_g(\dot{w} - uq + vp) - z_g(\dot{v} - wp + ur)] \\
\quad -(i + pq)I_{xx} + (r^2 - q^2)I_{yy} + (pr - \dot{q})I_{xy} \\
M = I_y \dot{q} + (I_x - I_z)rp + M_q q + M_{|q|q}|q|q + m[z_g(\dot{u} - vr + wq) - x_g(\dot{w} - uq + vp)] \\
\quad -(p + qr)I_{yy} + (p^2 - r^2)I_{zz} + (qp - \dot{r})I_{xz} \\
N = I_z \dot{r} + (I_y - I_x)pq + N_r r + N_{|r|r}|r|r + m[x_g(\dot{v} - wp + ur) - y_g(\dot{u} - vr + wq)] \\
\quad -(q + rp)I_{zz} + (p^2 - q^2)I_{yy} + (rq - \dot{p})I_{xz}
\end{cases}
\tag{1}
$$

**Supplementary Table 1. INERTIAL PARAMETERS of HAIXUN 2**

| Name | Unit | Value |
|---|---|---|
| $x_g$ | m | 0 |
| $y_g$ | m | 0 |
| $z_g$ | m | $-0.1$ |
| $x_b$ | m | 0 |
| $y_b$ | m | 0 |
| $z_b$ | m | 0.03 |
| $I_x$ | kg·m$^2$ | 24.832 |
| $I_y$ | kg·m$^2$ | 17.697 |
| $I_z$ | kg·m$^2$ | 24.928 |
| $I_{xx}$ | kg·m$^2$ | 27.220 |
| $I_{xy}$ | kg·m$^2$ | $-0.220$ |
| $I_{yy}$ | kg·m$^2$ | 34.371 |
| $I_{xz}$ | kg·m$^2$ | 4.110 |
| $I_{yz}$ | kg·m$^2$ | 0.462 |
| $I_{zz}$ | kg·m$^2$ | 24.903 |

**Supplementary Table 2. HYDRODYNAMIC PARAMETERS of HAIXUN 2**

| | | |
|---|---|---|
| $X_u = -268.2$ | $Y_v = -539.6$ | $Z_w = -112.9$ |
| $X_{\|u\|u} = 26.9$ | $Y_{\|v\|v} = 35.8$ | $Z_{\|w\|w} = 6.2$ |
| $K_p = -104.6$ | $M_q = -64.8$ | $N_r = -80.4$ |
| $K_{\|p\|p} = 3.0$ | $M_{\|q\|q} = 4.9$ | $N_{\|r\|r} = 3.5$ |

**Supplementary Table 3. CONTROLLER PARAMETERS**

| Name | Value |
|---|---|
| $\boldsymbol{Q}_1$ | $diag(10^2, 10^2, 10^1, 10^0, 10^{-1}, 10^2)$ |
| $\boldsymbol{P}_1$ | $diag(50, 50, 10^1, 10^{-1}, 10^{-1}, 10^0)$ |
| $\boldsymbol{R}_1$ | $diag(10^1, 10^1, 10^1, 20, 20, 10^1)$ |
| $\boldsymbol{Q}_2$ | $diag(50, 10^1, 10^1, 10^{-2}, 10^{-2}, 10^0)$ |
| $\boldsymbol{P}_2$ | $diag(50, 10^0, 10^0, 10^{-2}, 10^{-2}, 10^0)$ |
| $\boldsymbol{R}_2$ | $diag(10^{-4}, 10^{-4}, 10^{-4}, 10^{-4}, 10^{-4}, 10^{-4})$ |
| $\boldsymbol{K}_\mathrm{p}$ | $diag(10^2, 10^2, 10^1, 10^0, 10^{-1}, 10^2)$ |
| $\boldsymbol{K}_\mathrm{d}$ | $diag(5, 5, 5, 5, 5, 5)$ |
| $\boldsymbol{K}_1$ | $diag(10^2, 10^2, 10^1, 10^0, 10^{-1}, 10^2)$ |
| $\Lambda$ | 1 |
| $\Gamma$ | 1 |